



## ECONOMIC ASEISMIC DESIGN OF RIGID RETAINING WALL

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### Abstract

Rigid retaining walls experience significant displacements during earthquakes. Several investigators have developed 1-D and 2-D models to predict displacements. A critical review of the state of the art shows that these model may not predict realistic displacements Wu (1999).

A new 2-D model, which considers strain dependant soil stiffness and material damping, sliding and rocking motions, and practical field water conditions behind the wall as per Eurocode (1994) has been developed (Wu 1999). This model represents a considerable advance over the existing solutions and is easily useable by the practicing engineer. It has been shown that walls inclined on the back fill offer several technical advantages

### Introduction

The traditional design of a rigid retaining wall requires estimating the earth pressure behind a wall geometry that provides a sufficient factor of safety against sliding, rotation and bearing capacity of the wall. This method is known as the limit design method. Two classical earth pressure theories, Coulomb (1776) and Rankine (1857), have been applied to the static design of earth-retaining structures. For both theories, the movement of retaining walls is limited for the state of plastic equilibrium to mobilize and fully develop the active earth pressure. Therefore, limiting movements of retaining structures are expected in the static design.

For their design in seismically active regions, the limit design method is adopted as a basic concept, where the dynamic earth pressure is calculated by Mononobe-Okabe method for modified Coulomb's method. No displacements have been specified for developing fully active conditions. However, large scale tests with cohesionless backfills have shown that the horizontal pressure is highly dependant on the magnitude (a top deflection of 0.003 height of the wall) and direction of wall movement (USCOE No. 4, 1994). Hence, some displacements are expected to take place in both static and dynamic conditions and, more specifically, the earth pressure on the structure is related to the magnitude of displacement.

However, this method does not necessarily provide a safe estimation of displacements for structures subjected to dynamic loading during earthquakes. The displacements that remain within an acceptable limit may not be assured. Therefore, the movements of retaining structures during earthquakes may cause severe damage to the retaining walls or to the adjoining structures.

A detailed summary of retaining wall displacements and damages during earthquakes has been reported Seed and Whitman (1970), Shakya (1987),

Prakash et al. (1995) and Iai (1998), Wu (1999) and Wu and Prakash (2001). It has been shown that the rigid retaining walls experience both sliding and rotation. Wu (1999) reviewed the available models and concluded that these are not sufficient to predict credible displacements. Wu and Prakash (2001) described their model for computation of displacements of vertical and inclined rigid walls. A detailed study of vertical and inclined walls shows that walls inclined towards the back fill offer several technical advantages and economical section.

### DESCRIPTION OF MODEL

Rafnsson (1991) developed a model for simulating the response of rigid retaining walls. This model consisted of a rigid wall resting on the foundation soil and subjected to a horizontal ground motion. Both material and geometrical damping in sliding and rocking motions were considered, Figure 1, (Rafnsson and Prakash 1994).

The mathematical model in Figure 2 represents the displacements in active case. Soil nonlinearity is included in defining the following properties, both at the base and backfill:

- (1) soil stiffness in sliding,
- (2) soil stiffness in rocking,
- (3) geometrical damping in sliding,
- (4) geometrical damping in rocking,
- (5) material damping in sliding,
- (6) material damping in rocking.

### WU'S (1999) MODEL

Rafnsson (1991) considered only dry soil and the real ground motion was idealized as equivalent sinusoidal motion of arbitrarily selected frequencies. Also, the backfill soil had been simulated as an active spring.

Wu (1999) modified Rafnsson model by considering dry and submerged soils and the walls subjected to real ground motion with nonlinear soil properties.

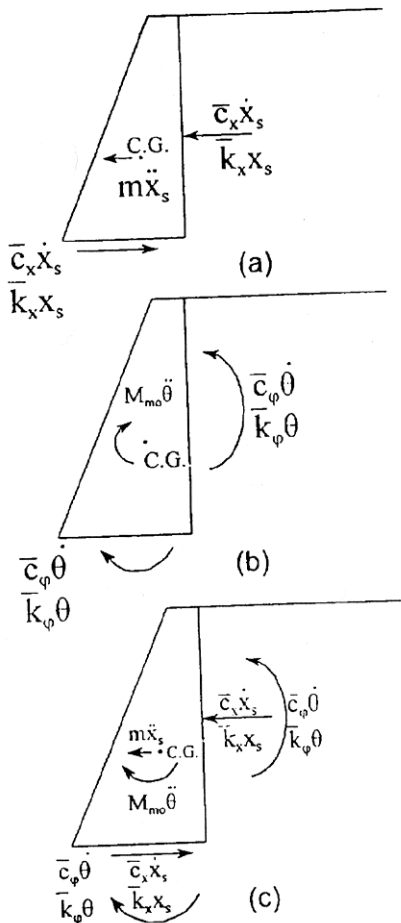


Figure 1. System of forces in mathematical model of retaining wall: a) sliding only, b) rocking only c) combining sliding and rocking (Rafnsson 1991, Rafnsson and Prakash 1994)

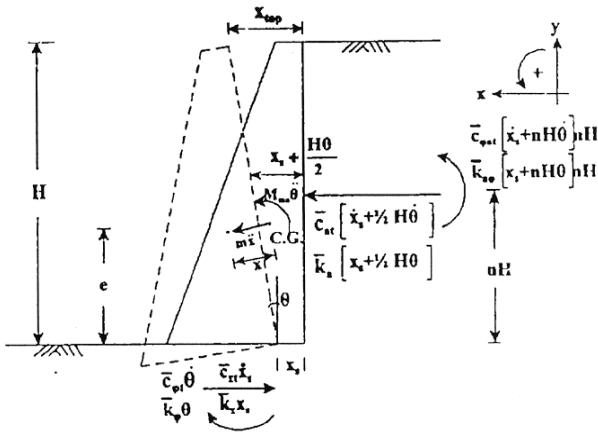


Figure 2 Mathematical Model for stiffness and damping constants for the active case (Rafnsson 1991, Rafnsson and Prakash 1994)

Also, backfill force was represented by a time dependant active force. These present a considerable advance in the analysis. Also both vertical and inclined walls have been considered. For details see Wu (1999) and Wu and Prakash (2001).

### SOIL – WALL INTERACTION PROPERTIES

In Wu's model Figures 3 and 4, the resistance from the foundation soil is represented by the stiffness, and damping values of the foundation soil. The stiffness and geometrical damping values are directly dependent on the shear modulus of the soil. Furthermore, both the shear modulus and the material damping are strain dependent. Other factors that need to be evaluated are the Poisson's ratio, soil density, void ratio, plasticity index and the shear strain that the soil experiences during earthquakes.

Retaining walls are subjected to soil reactions and damping at the foundation soil. Realistically, the following parameters must be determined at the foundation:

- (1) soil stiffness in sliding and rocking of the foundation soil,
- (2) damping in sliding and rocking of the foundation,

Note that, damping values include material and geometric damping.

For details, see Wu (1999), and Wu and Prakash (2001). Wu (1999) studied the walls inclined by 0 to  $\pm 5^\circ$  with the vertical, both away and towards the fill (Table 1). The reference wall, with nine different incline angles ( $0^\circ, 1.25^\circ, 2.5^\circ, 3.75^\circ, +5^\circ, -1.25^\circ, -2.5^\circ, -3.75^\circ,$  and  $-5^\circ$ ) subjected to Northridge earthquake condition was used. Figure 5 showed the computed displacements of this wall with  $0^\circ, +5^\circ$  and  $-5^\circ$  incline angles at the back. The negative angle at the back of the wall is the case of the wall resting on the backfill. Table 1 shows a summary of new base widths and computed displacement for various incline angles.

Table 1. Cumulative displacement for three incline angle of inclination at the back of a wall subjected to Northridge earthquake condition (B=3.57m).

Inclination angle (degree)	Base width (m)	Cumulative Displacement by Fixed Base Width (3.57m)			
		Sliding (m)	Rocking (degree)	Rocking (m)	Total (m)
+5.00°	3.81	0.0820	1.31	0.1374	0.2194
+3.75°	3.76	0.0820	1.30	0.1366	0.2186
+2.50°	3.70	0.0815	1.30	0.1361	0.2176
+1.25°	3.63	0.0808	1.29	0.1355	0.2163
0.00°	3.57	0.0808	1.29	0.1347	0.2155
-1.25°	3.50	0.0806	1.28	0.1338	0.2144
-2.50°	3.43	0.0805	1.27	0.1329	0.2134
-3.75°	3.35	0.0803	1.26	0.1320	0.2123
-5.00°	3.38	0.0801	1.25	0.1311	0.2112

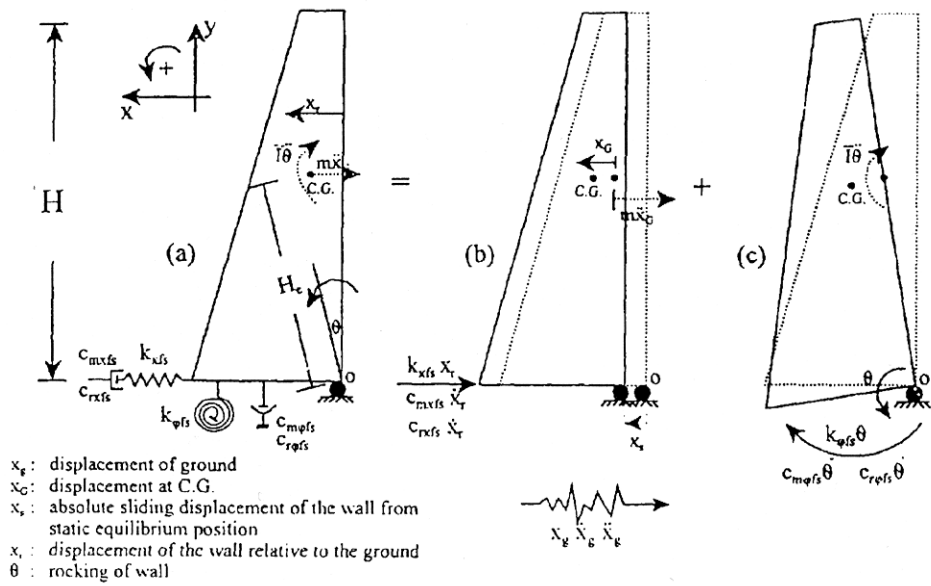


Figure 3. Two degree of freedom system a) rigid retaining wall with spring and dashpots, b) free body diagram of sliding and c) free body diagram of rocking (After Wu, 1999)

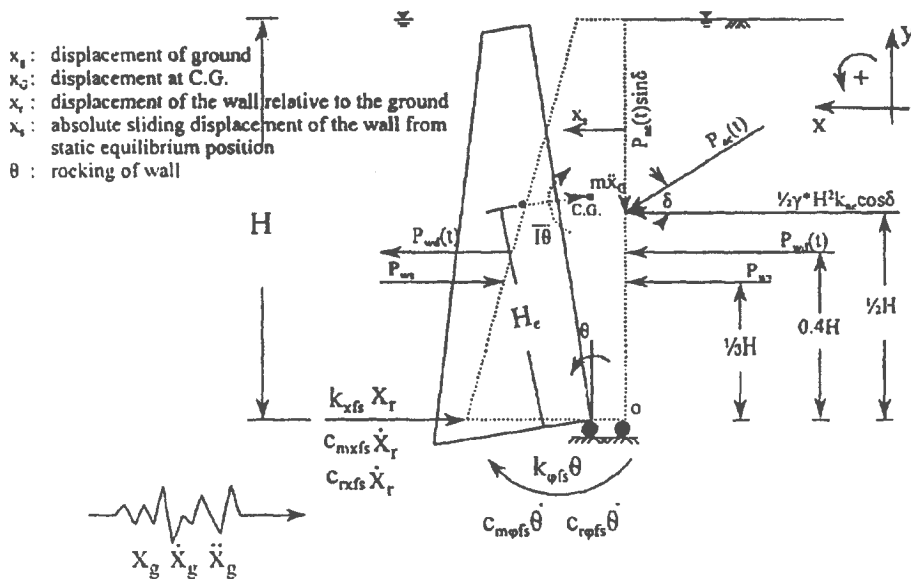


Figure 4. Force diagram of forced vibration of rigid retaining wall with submerged pervious backfill (After Wu, 1999)

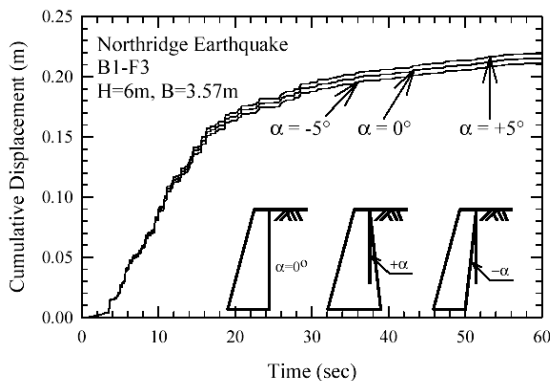


Figure 5. Cumulative displacements of walls (B1-F3) with different inclinations with the vertical (Wu 1999)

The computed cumulative sliding, rocking and total displacements are also shown in this table. The base widths decreased from 3.57m to 3.38m as the inclination changed from 0° to -5°, since the active earth forces decrease with negative inclination. Therefore, the base width was smaller for a wall with a negative inclination.

The rocking degrees (Table 1) decreased from 1.29° (α=0) to 1.25° (α = -5°), and the total displacements decreased slightly from 0.2155m to 0.2112m. The cumulative displacements for these walls will not be significantly altered by changing the incline angle at the back of the wall.

For the wall built as a leaning-type rigid retaining wall with -5° lying on the backfill, the wall experienced a

rocking movement of 1.25° during the Northridge earthquake. Therefore, when the wall was subjected to the same earthquake event up to 3 or 4 times, the wall experienced a total rocking degree close to 5°. At this time, the wall may become vertical.

### Results and Discussions:

Wu (1999) had studied 21 – backfill and foundations soil combination (Table 2). Typical results of a reference wall 6m high, subjected to Northridge earthquake are listed in Table 3.

It will be seen that wall with B3-F7 experiences the largest rotation of 2.81° and cumulative displacement of 0.4129m for α = -5°. If this wall experiences shocks similar to Northridge, it may become vertical in 2-events.

Also its base width with α = 0 is 2.89m while with α = -5°, it is less than 2.89m.

Figure 6 shows plot of cumulative displacements of wall B3-F7 with α=0, -2° and -5°. It will be seen that cumulative displacements are not altered for any practical purpose.

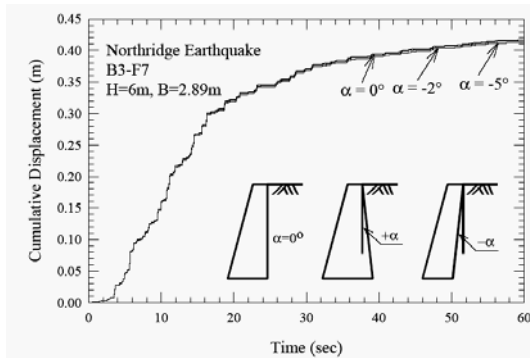


Figure 6. . Cumulative displacements of walls (B3-F7) with different inclinations with the vertical (Wu 1999)

Table 4 shows design widths of foundations for 21 cases. It will be seen that for B3-F7, the base width reduces from 2.89 ( $\alpha = 0^\circ$ ) to 2.62 ( $\alpha = -5^\circ$ ). This results in savings of 9.3% in the volume of the wall. For other cases, the savings vary from 8-10%.

### Conclusions:

The following conclusions may be drawn:

1. A wall with negative inclination is technically a sound proposition during earthquakes. It may stand 2-3 shocks before it becomes vertical and may prevent overturning.
2. These walls will be economical in section to about 10% in the volume of the material.

### Acknowledgements:

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Table 2 Engineering properties for both foundation soil and backfill (Naval, 1986).

FOUNDATION SOIL (F)

	Soil Type	$\gamma_d$ kN/m <sup>3</sup>	N deg	* degree	void ratio	<	C kN/m <sup>2</sup>	PI	w%
F-1	GW	21.07	37.5	25.0	0.25	0.3	-	-	6
F-2	GP	19.18	36.0	24.0	0.36	0.3	-	-	6
F-3	SW	18.00	35.0	23.3	0.46	0.3	-	-	8
F-4	SP	16.82	34.0	22.7	0.56	0.3	-	-	10
F-5	SM	15.70	33.0	22.0	0.68	0.3	-	4	15
F-6	SC	14.00	30.0	20.0	0.88	0.3	-	13	25
F-7	ML	14.15	32.0	21.3	0.85	0.3	9.57	4	14

BACKFILL (B)

	Soil Type	$\gamma_d$ kN/m <sup>3</sup>	N degree	* degree	void ratio	<	C kN/m <sup>2</sup>	PI	W%
B-1	GM	19.6	33.0	22.0	0.35	0.3	-	-	10
B-2	GP	18.9	34.0	22.7	0.40	0.3	-	-	8
B-3	SP	15.6	34.0	22.7	0.69	0.3	-	-	8

- All properties for backfill are for the condition of 90 percent of the "Standard Proctor".

Table 3. Cumulative Displacement<sup>2</sup> of 6m High Wall at Different Inclination Angles Subjected to Northridge Earthquake

Soil Comb.	Base Width (m)	Vertical (0 degree)			-1 degree			-2 degree			-3 degree			-4 degree			-5 degree		
		Sliding	Rocking	Total	Sliding	Rocking	Total	Sliding	Rocking	Total	Sliding	Rocking	Total	Sliding	Rocking	Total	Sliding	Rocking	Total
		m	degree (m)	m	m	degree (m)	m	m	degree (m)	m	m	degree (m)	M	m	degree (m)	m	m	degree (m)	m
B1-F1	3.22	0.0809	1.54 (0.1615)	0.2424	0.0807	1.53 (0.1605)	0.2413	0.0807	1.53 (0.1598)	0.2404	0.0805	1.52 (0.1591)	0.2396	0.0803	1.51 (0.1583)	0.2386	0.0802	1.50 (0.1576)	0.2377
B1-F2	3.42	0.0895	1.62 (0.1690)	0.2585	0.0895	1.61 (0.1981)	0.2576	0.0894	1.60 (0.1674)	0.2568	0.0893	1.59 (0.1667)	0.256	0.0892	1.58 (0.1660)	0.2552	0.0891	1.58 (0.1652)	0.2543
B1-F3	3.57	0.0808	1.29 (0.1347)	0.2155	0.0807	1.28 (0.1339)	0.2146	0.0806	1.27 (0.1332)	0.2138	0.0805	1.27 (0.1325)	0.2130	0.0803	1.26 (0.1319)	0.2121	0.0801	1.25 (0.1311)	0.2112
B1-F4	3.71	0.0893	1.39 (0.1457)	0.2350	0.0890	1.38 (0.1446)	0.2336	0.0889	1.38 (0.1438)	0.2327	0.0888	1.37 (0.1431)	0.2319	0.0887	1.36 (0.1424)	0.2311	0.0886	1.35 (0.1416)	0.2302
B1-F5	4.07	0.0944	1.35 (0.1414)	0.2358	0.0938	1.34 (0.1405)	0.2343	0.0938	1.34 (0.1398)	0.2336	0.0937	1.33 (0.1391)	0.2328	0.0936	1.32 (0.1384)	0.2320	0.0935	1.31 (0.1376)	0.2311
B1-F6	4.61	0.1035	1.36 (0.1421)	0.2456	0.1035	1.35 (0.1417)	0.2451	0.1034	1.35 (0.1411)	0.2445	0.1034	1.34 (0.1404)	0.2438	0.1033	1.33 (0.1397)	0.2430	0.1033	1.33 (0.1390)	0.2423
B1-F7	4.05	0.1190	1.89 (0.1975)	0.3165	0.1190	1.88 (0.1970)	0.3160	0.1191	1.87 (0.1963)	0.3154	0.1191	1.87 (0.1956)	0.3147	0.1190	1.86 (0.1949)	0.3139	0.1190	1.85 (0.1941)	0.3131
B2-F1	2.85	0.0826	1.88 (0.1971)	0.2797	0.0823	1.87 (0.1963)	0.2787	0.0823	1.87 (0.1955)	0.2778	0.0821	1.86 (0.1947)	0.2769	0.0820	1.85 (0.1938)	0.2759	0.0819	1.84 (0.1930)	0.2748
B2-F2	3.13	0.0896	1.82 (0.1906)	0.2802	0.0896	1.81 (0.1900)	0.2790	0.0895	1.81 (0.1892)	0.2787	0.0894	1.80 (0.1884)	0.2778	0.0893	1.79 (0.1876)	0.2769	0.0892	1.78 (0.1867)	0.2759
B2-F3	3.17	0.0836	1.59 (0.1660)	0.2496	0.0834	1.58 (0.1651)	0.2485	0.0833	1.57 (0.1643)	0.2476	0.0832	1.56 (0.1635)	0.2467	0.0831	1.55 (0.1626)	0.2457	0.0829	1.55 (0.1618)	0.2447
B2-F4	3.39	0.0900	1.59 (0.1665)	0.2565	0.0899	1.58 (0.1659)	0.2558	0.0898	1.58 (0.1651)	0.2549	0.0897	1.57 (0.1643)	0.2540	0.0892	1.56 (0.1638)	0.2530	0.0891	1.56 (0.1629)	0.2520
B2-F5	3.74	0.0946	1.53 (0.1600)	0.2546	0.0941	1.52 (0.1591)	0.2532	0.0940	1.51 (0.1584)	0.2524	0.0939	1.51 (0.1577)	0.2516	0.0938	1.50 (0.1569)	0.2507	0.0937	1.49 (0.1561)	0.2498
B2-F6	4.23	0.1047	1.53 (0.1598)	0.2645	0.1038	1.53 (0.1598)	0.2637	0.1038	1.52 (0.1591)	0.2629	0.1038	1.52 (0.1584)	0.2622	0.1037	1.51 (0.1577)	0.2614	0.1037	1.50 (0.1569)	0.2606
B2-F7	3.61	0.1205	2.23 (0.2334)	0.3539	0.1205	2.22 (0.2321)	0.3525	0.1205	2.21 (0.2314)	0.3518	0.1205	2.20 (0.2306)	0.3511	0.1204	2.19 (0.2298)	0.3502	0.1204	2.19 (0.2289)	0.3493
B3-F1	2.27	0.0808	2.52 (0.2635)	0.3443	0.0807	2.50 (0.2618)	0.3425	0.0806	2.49 (0.2610)	0.3417	0.0805	2.48 (0.2602)	0.3408	0.0804	2.48 (0.2593)	0.3398	0.0803	2.47 (0.2584)	0.3387
B3-F2	2.52	0.0876	2.39 (0.2500)	0.3376	0.0875	2.37 (0.2486)	0.3361	0.0874	2.37 (0.2479)	0.3353	0.0874	2.36 (0.2471)	0.3344	0.0873	2.35 (0.2463)	0.3335	0.0871	2.34 (0.2454)	0.3325
B3-F3	2.63	0.0822	2.02 (0.2118)	0.2940	0.0821	2.02 (0.2107)	0.2929	0.0821	2.01 (0.2100)	0.2920	0.0820	2.00 (0.2092)	0.2911	0.0818	1.99 (0.2083)	0.2901	0.0817	1.98 (0.2075)	0.2891
B3-F4	2.92	0.0868	1.87 (0.1955)	0.2823	0.0867	1.86 (0.1948)	0.2815	0.0867	1.85 (0.1940)	0.2807	0.0866	1.85 (0.1933)	0.2799	0.0865	1.84 (0.1924)	0.2790	0.0864	1.83 (0.1916)	0.2780
B3-F5	3.16	0.0931	1.88 (0.1967)	0.2898	0.0924	1.86 (0.1949)	0.2873	0.0922	1.86 (0.1944)	0.2873	0.0921	1.85 (0.1936)	0.2857	0.0921	1.84 (0.1928)	0.2849	0.0919	1.83 (0.1921)	0.2840
B3-F6	4.56	0.0917	1.18 (0.1240)	0.2159	0.0916	1.18 (0.1235)	0.2151	0.0916	1.17 (0.1228)	0.2144	0.0916	1.17 (0.1222)	0.2137	0.0915	1.16 (0.1215)	0.2130	0.0915	1.15 (0.1207)	0.2122
B3-F7	2.89	<b>0.1184</b>	<b>2.85</b> <b>(0.2983)</b>	<b>0.4167</b>	0.1184	2.84 (0.2975)	0.4159	0.1185	2.83 (0.2969)	0.4153	0.1185	2.83 (0.2961)	0.4146	0.1185	2.82 (0.2953)	0.4138	<b>0.1184</b>	<b>2.81</b> <b>(0.2945)</b>	<b>0.4129</b>

<sup>1</sup> Base width computed from 0 inclination angle, <sup>2</sup> Displacement computed with fixed base width shown in the second column

Table 4. Foundation widths of 21-walls for static forces

Soil Comb.	Base Width (m)					
	Vertical	-1 degree	-2 degree	-3 degree	-4 degree	-5 degree
B1-F1	3.22	3.17	3.12	3.17	3.12	2.96
B1-F2	3.42	3.47	3.31	3.26	3.20	3.24
B1-F3	3.57	3.51	3.46	3.40	3.34	3.38
B1-F4	3.71	3.65	3.69	3.63	3.66	3.60
B1-F5	4.07	4.01	4.05	3.98	3.92	3.95
B1-F6	4.61	4.64	4.57	4.49	4.52	4.44
B1-F7	4.05	3.99	3.92	3.85	3.78	3.71
B2-F1	2.85	2.80	2.76	2.71	2.76	2.61
B2-F2	3.13	2.98	2.93	2.88	2.93	2.77
B2-F3	3.17	3.12	3.06	3.01	2.95	2.99
B2-F4	3.39	3.34	3.28	3.32	3.26	3.30
B2-F5	3.74	3.69	3.63	3.67	3.6	3.54
B2-F6	4.23	4.16	4.10	4.12	4.05	3.98
B2-F7	3.61	3.64	3.58	3.42	3.35	3.38
B3-F1	2.27	2.23	2.19	2.15	2.11	2.07
B3-F2	2.52	2.38	2.43	2.29	2.25	2.20
B3-F3	2.63	2.59	2.44	2.40	2.45	2.40
B3-F4	2.93	2.79	2.74	2.69	2.74	2.69
B3-F5	3.16	3.01	2.96	3.01	2.96	3.00
B3-F6	4.56	4.60	4.55	4.49	4.53	4.47
B3-F7	2.89	2.84	2.79	2.83	2.68	2.62